B.Sc (NM)/B.A – I (Sem-I)

Linear Algebra

1. Elementary Operations on a matrix

There are six operations on a matrix, three on rows called Elementary row operations and three on columns called Elementary column operations or transformations of a matrix.

1.) The interchange of any two rows or columns.

2.) The multiplication of all the elements of any row or column by any non-zero number.

3.) The addition of scalar multiple of any row (column) to the corresponding elements of any other row (column).

Symbols used for the above transformations are given below respectively:

1.) $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$ means interchange of ith and jth row (column).

2.) $R_i \to kR_i$ or $C_i \to kC_i$ means multiplication of ith row (column) by a non-zero constant k.

3.) $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$ means addition of ith row ~~col~~ (column) to the addition of k times jth row (column).

2.) Inverse of matrix using Gauss Jordan Method

Gauss-Jordan method is a variant of Gaussian elimination in which row reduction operation is performed to find the inverse of a matrix. Steps to find the inverse of a matrix using Gauss-Jordan method:

Step I : Write $A = IA$ or $A = AI$.

Step II : Perform the row reduction operations or column reduction operations successively on A on L.H.S. and ~~and~~ on the pre-factor I or Post-factor I of R.H.S. till we get $I = BA$ or $I = AB$.

Then we get $B = A^{-1}$

Note: When applying elementary row or column operations, If we get all zeros in a row (column) of the matrix A on L.H.S., then A^{-1} does not exist.

Example 1.) Using Gauss-Jordan method, find inverse, if exists, of the following matrices:

(i) $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Solution: (i) let $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ then

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [\text{Applying } R_1 \to R_1 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \quad [\text{Applying } R_2 \to R_2 - 2R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \quad [\text{Applying } R_1 \to R_1 - 3R_2]$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}.$$

(ii) let $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ then

$$A = AI$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} = A \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} \quad [\text{Applying } C_2 \to C_2 - \tfrac{1}{2} C_1]$$

Since there are all zeros in the second column of the L.H.S. matrix of above equation, therefore A^{-1} does not exist.

(iii) let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ then

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} A \quad [R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} A \quad [R_1 \to R_1 + 2R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} A \quad [R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & \frac{3}{2} \end{bmatrix} A \quad [R_2 \to \tfrac{1}{2} R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}.$$

Example 2. Find the inverse of the matrix

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$ with the help of elementary row operations.

Solution: let $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ then

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$